

## Summer 2021

### Algebra 1/1H Summer Packet

The goal of summer math is to ensure that students are prepared for high school math classes. The skills learned in elementary and middle school are an integral part of success at the high school level, and this packet covers many of the essential skills that students entering high school should have mastered.

All students entering Algebra 1 or Algebra 1 Honors must complete this packet before school starts. On the first day of school, you will be asked to sign an Honor Code statement that you completed this packet, but it will not be collected. **You will be tested on the concepts in this packet during the first or second week of school as your first grade for the year.**

**You only need to do the circled problems on each page.** The expectation throughout Algebra 1 is that you attempt every problem given and show your work neatly. Use loose leaf paper and clearly number each topic and problem if there is not enough work space on the packet pages.

**Calculators are not permitted when working on this packet and will not be permitted for at least the first quarter of the school year.**

Each page of the packet summarizes a concept with examples. Use those examples if you have trouble remembering how to do any of the problems. **We recommend the following websites if you need more help with any of the topics:**

[www.purplemath.com](http://www.purplemath.com)  
[www.khanacademy.org](http://www.khanacademy.org)  
[www.mathbitsnotebook.com](http://www.mathbitsnotebook.com)  
[www.mathantics.com](http://www.mathantics.com)

**For Algebra 1 and Algebra 1 Honors, the following materials will be required:**

- Graphing calculator (TI-84 Plus or TI-84 Plus CE)
- Two (2) separate notebooks, either spiral or composition (One will be for notes and one will be for homework. They will be collected separately from time to time for grading)
- Two-pocket folder
- Computer stylus (issued by school with your laptop and must be replaced with the same model if it is lost or broken)
- Spare AAAA battery for the stylus at all times
- Erasable pens in black, blue, and red (Frixion pens recommended)
- Pencils
- Highlighters (2 colors)
- 6-inch ruler
- Graph paper
- Loose leaf paper
- Two thin and two thick Expo markers of different colors at all times

**Good luck, pace yourself, and have a great summer!**

# Prime Factorization

Because 3 is a factor of 24 and  $3 \cdot 8 = 24$ , 8 is also a factor of 24. The pair 3, 8 is called a **factor pair** of 24.

The **prime factorization** of a composite number is the number written as a product of its prime factors. You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product.

**Example 1** A classroom has 42 students. The teacher arranges the students in rows. Each row has the same number of students. How many possible arrangements are there?

Use the factor pairs of 42 to find the number of arrangements.

$$42 = 1 \cdot 42 \quad 1 \text{ row of 42 or 42 rows of 1} \quad 42 = 2 \cdot 21 \quad 2 \text{ rows of 21 or 21 rows of 2}$$

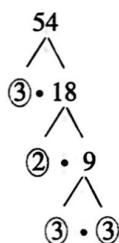
$$42 = 3 \cdot 14 \quad 3 \text{ rows of 14 or 14 rows of 3} \quad 42 = 6 \cdot 7 \quad 6 \text{ rows of 7 or 7 rows of 6}$$

► There are 8 possible arrangements: 1 row of 42, 42 rows of 1, 2 rows of 21, 21 rows of 2, 3 rows of 14, 14 rows of 3, 6 rows of 7, or 7 rows of 6.

**Example 2** Write the prime factorization of 54.

Choose any factor pair of 54 to begin the factor tree.

**Tree 1**



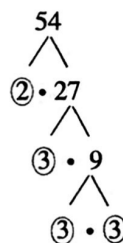
$$54 = 3 \cdot 2 \cdot 3 \cdot 3$$

Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

**Tree 2**



$$54 = 2 \cdot 3 \cdot 3 \cdot 3$$

► The prime factorization of 54 is  $2 \cdot 3 \cdot 3 \cdot 3$ , or  $2 \cdot 3^3$ .

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

List the factor pairs of the number.

1. 16

2. 30

③. 63

4. 100

⑤. 135

6. 275

Write the prime factorization of the number.

7. 24

8. 66

9. 50

⑩. 80

11. 98

12. 126

13. 154

⑭. 310

Find the greatest perfect square that is a factor of the number.

15. 117

16. 150

17. 539

⑮. 936

19. **EXERCISE** An exercise class has 28 participants. The instructor arranges the participants in rows. Each row has the same number of participants. How many possible arrangements are there?

# Greatest Common Factor

Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor (GCF)**. There are several different ways to find the GCF of two or more numbers.

**Example 1 Find the greatest common factor (GCF) of 56 and 104.**

**Method 1** List the factors of each number. Then circle the common factors.

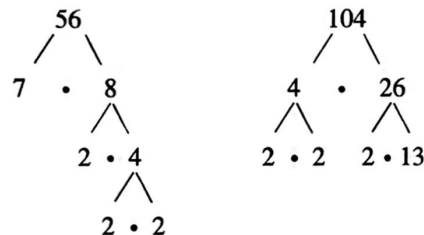
**Factors of 56:** ①, ②, ④, 7, ⑧, 14, 28, 56

**Factors of 104:** ①, ②, ④, ⑧, 13, 26, 52, 104

The common factors are 1, 2, 4, and 8. The greatest of these common factors is 8.

► So, the GCF of 56 and 104 is 8.

**Method 2** Make a factor tree for each number.



Write the prime factorization of each number. Then circle the common prime factors. The GCF is the product of the common prime factors.

$$56 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot 7$$

$$104 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot 13$$

► So, the GCF of 56 and 104 is  $2 \cdot 2 \cdot 2 = 8$ .

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

**Find the GCF of the numbers using the two methods shown above.**

1. 30, 45

② 12, 54

3. 16, 96

4. 42, 98

5. 27, 66

6. 50, 160

7. 21, 70

⑧ 76, 95

9. 60, 84

10. 60, 120, 210

11. 44, 64, 100

⑫ 15, 28, 70

13. Write a set of two numbers that have a GCF of 20. Explain how you found your answer.

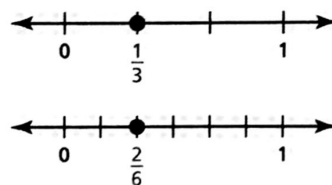
⑭ Write a set of three numbers that have a GCF of 25. Explain how you found your answer.

⑮ **BOUQUETS** A florist is making identical bouquets using 90 white roses, 60 red roses, and 45 pink roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?

16. **FABRIC** You have two pieces of fabric. One piece is 6 feet wide and the other piece is 7.5 feet wide. You want to cut both pieces into strips of equal width that are as wide as possible. How wide should you cut the strips of fabric?

# Equivalent Fractions and Simplifying Fractions

The number lines show the graphs of two fractions,  $\frac{1}{3}$  and  $\frac{2}{6}$ . These fractions represent the same number. Two fractions that represent the same number are called **equivalent fractions**. To write equivalent fractions, you can multiply or divide the numerator and the denominator by the same nonzero number.



**Example 1** Write two fractions that are equivalent to  $\frac{8}{12}$ .

Multiply the numerator and denominator by 2.

$$\frac{8}{12} = \frac{8 \cdot 2}{12 \cdot 2} = \frac{16}{24}$$

Divide the number and denominator by 2.

$$\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

► Two equivalent fractions are  $\frac{16}{24}$  and  $\frac{4}{6}$ .

A fraction is in **simplest form** when its numerator and its denominator have no common factors besides 1.

**Example 2** Write the fraction  $\frac{18}{24}$  in simplest form.

Divide the numerator and denominator by 6, the greatest common factor of 18 and 24.

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

►  $\frac{18}{24}$  in simplest form is  $\frac{3}{4}$ .

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Write two fractions that are equivalent to the given fraction.

1.  $\frac{4}{10}$

2.  $\frac{3}{7}$

3.  $\frac{10}{15}$

4.  $\frac{16}{20}$

5.  $\frac{9}{30}$

6.  $\frac{1}{8}$

7.  $\frac{9}{16}$

8.  $\frac{12}{14}$

Write the fraction in simplest form.

9.  $\frac{18}{27}$

10.  $\frac{3}{18}$

11.  $\frac{35}{50}$

12.  $\frac{14}{32}$

13.  $\frac{4}{36}$

14.  $\frac{48}{80}$

15.  $\frac{24}{63}$

16.  $\frac{33}{88}$

17.  $\frac{45}{100}$

18.  $\frac{60}{150}$

19.  $\frac{48}{96}$

20.  $\frac{110}{170}$

21. Is the fraction  $\frac{45}{61}$  in simplest form? Explain.

22. Write five fractions that each simplify to one-ninth.

23. **SLEEP** It is recommended that 10- to 17-year old students should sleep about 9 hours each night. What fraction of the day is this? Write your answer in simplest form.



# Adding and Subtracting Fractions

To add or subtract two fractions with *like denominators*, write the sum or difference of the numerators over the denominator.

## Adding or Subtracting Fractions with Like Denominators

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}, \text{ where } c \neq 0 \quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}, \text{ where } c \neq 0$$

**Example 1** Find  $\frac{7}{12} + \frac{1}{12}$ .

$$\begin{aligned} \frac{7}{12} + \frac{1}{12} &= \frac{7+1}{12} && \text{Add the numerators.} \\ &= \frac{8}{12}, \text{ or } \frac{2}{3} && \text{Simplify.} \end{aligned}$$

**Example 2** Find  $\frac{7}{9} - \frac{2}{9}$ .

$$\begin{aligned} \frac{7}{9} - \frac{2}{9} &= \frac{7-2}{9} && \text{Subtract the numerators.} \\ &= \frac{5}{9} && \text{Simplify.} \end{aligned}$$

To add or subtract two fractions with *unlike denominators*, first write equivalent fractions with a common denominator. There are two methods you can use.

## Adding or Subtracting Fractions with Unlike Denominators

**Method 1** Multiply the numerator and the denominator of each fraction by the denominator of the other fraction.

**Method 2** Use the **least common denominator (LCD)**. The LCD of two or more fractions is the least common multiple (LCM) of the denominators.

**Example 3** Find  $\frac{1}{8} + \frac{5}{6}$ .

**Method 1:**  $\frac{1}{8} + \frac{5}{6} = \frac{1 \cdot 6}{8 \cdot 6} + \frac{5 \cdot 8}{6 \cdot 8}$  Rewrite using a common denominator of  $8 \cdot 6 = 48$ .

$$\begin{aligned} &= \frac{6}{48} + \frac{40}{48} && \text{Multiply.} \\ &= \frac{46}{48}, \text{ or } \frac{23}{24} && \text{Simplify.} \end{aligned}$$

**Example 4** Find  $5\frac{3}{4} - 1\frac{7}{10}$ .

**Method 2:** Rewrite the difference as  $\frac{23}{4} - \frac{17}{10}$ .  
The LCM of 4 and 10 is 20. So, the LCD is 20.

$$\begin{aligned} \frac{23}{4} - \frac{17}{10} &= \frac{23 \cdot 5}{4 \cdot 5} - \frac{17 \cdot 2}{10 \cdot 2} && \text{Rewrite using the LCD, 20.} \\ &= \frac{115}{20} - \frac{34}{20} && \text{Multiply.} \\ &= \frac{81}{20}, \text{ or } 4\frac{1}{20} && \text{Simplify.} \end{aligned}$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

**Evaluate.**

1.  $\frac{1}{14} + \frac{5}{14}$

2.  $\frac{2}{5} + \frac{1}{5}$

3.  $\frac{9}{10} - \frac{1}{10}$

4.  $\frac{11}{16} - \frac{3}{16}$

5.  $\frac{5}{8} + \frac{7}{8}$

6.  $\frac{1}{6} + \frac{1}{6}$

7.  $\frac{7}{9} + \frac{2}{3}$

8.  $\frac{3}{5} + \frac{4}{7}$

9.  $\frac{3}{4} - \frac{1}{6}$

10.  $\frac{7}{12} - \frac{5}{9}$

11.  $\frac{9}{10} - \frac{5}{6}$

12.  $\frac{5}{12} + \frac{11}{16}$

13.  $2\frac{3}{5} + 1\frac{2}{5}$

14.  $4\frac{6}{7} - 2\frac{4}{7}$

15.  $5\frac{5}{12} + 3\frac{3}{8}$

16.  $8\frac{1}{3} - 3\frac{2}{11}$

17.  $\frac{1}{2} + 3\frac{2}{9}$

18.  $4\frac{3}{14} - \frac{1}{7}$

19.  $\frac{2}{7} + \frac{3}{4} + \frac{1}{2}$

20.  $\frac{13}{16} - \frac{1}{4} - \frac{3}{8}$

21.  $2\frac{1}{6} - \frac{5}{9} + \frac{2}{3}$

# Multiplying and Dividing Fractions

To multiply two fractions, multiply the numerators and multiply the denominators.

## Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \text{ where } b, d \neq 0$$

Example 1 Find  $\frac{2}{5} \cdot \frac{3}{8}$ .

$$\begin{aligned} \frac{2}{5} \cdot \frac{3}{8} &= \frac{2 \cdot 3}{5 \cdot 8} && \text{Multiply the numerators.} \\ &= \frac{2 \cdot 3}{5 \cdot 8} && \text{Multiply the denominators.} \\ &= \frac{\cancel{2} \cdot 3}{8 \cdot \cancel{4}} && \text{Divide out common factors.} \\ &= \frac{3}{20} && \text{Simplify.} \end{aligned}$$

Example 2 Find  $5\frac{1}{2} \cdot \frac{3}{4}$ .

$$\begin{aligned} 5\frac{1}{2} \cdot \frac{3}{4} &= \frac{11}{2} \cdot \frac{3}{4} && \text{Rewrite } 5\frac{1}{2} \text{ as } \frac{11}{2}. \\ &= \frac{11 \cdot 3}{2 \cdot 4} && \text{Multiply the numerators.} \\ &= \frac{33}{8}, \text{ or } 4\frac{1}{8} && \text{Multiply the denominators.} \end{aligned}$$

Two numbers whose product is 1 are **reciprocals**. To write the reciprocal of a number, write the number as a fraction. Then invert the fraction. Every number except 0 has a reciprocal.

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

## Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \text{ where } b, c, d \neq 0$$

Example 3 Find  $\frac{3}{7} \div \frac{5}{6}$ .

$$\begin{aligned} \frac{3}{7} \div \frac{5}{6} &= \frac{3}{7} \cdot \frac{6}{5} && \text{Multiply by the reciprocal} \\ &= \frac{3 \cdot 6}{7 \cdot 5} && \text{of } \frac{5}{6}, \text{ which is } \frac{6}{5}. \\ &= \frac{18}{35} && \text{Multiply.} \end{aligned}$$

Example 4 Find  $8 \div 2\frac{1}{3}$ .

$$\begin{aligned} 8 \div 2\frac{1}{3} &= 8 \div \frac{7}{3} && \text{Rewrite } 2\frac{1}{3} \text{ as } \frac{7}{3}. \\ &= 8 \cdot \frac{3}{7} && \text{Multiply by the reciprocal} \\ &= \frac{8 \cdot 3}{7} && \text{of } \frac{7}{3}, \text{ which is } \frac{3}{7}. \\ &= \frac{24}{7}, \text{ or } 3\frac{3}{7} && \text{Multiply.} \end{aligned}$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Write the reciprocal of the number. Simplify your answer if possible.

1.  $\frac{3}{8}$

2. 7

3. -12

4.  $-\frac{6}{5}$

Evaluate.

5.  $\frac{3}{4} \cdot \frac{1}{6}$

6.  $\frac{3}{10} \cdot \frac{2}{3}$

7.  $\frac{4}{9} \cdot \frac{2}{9}$

8.  $\frac{5}{8} \cdot \frac{7}{12}$

9.  $4 \cdot \frac{3}{16}$

10.  $3\frac{1}{2} \cdot \frac{6}{7}$

11.  $1\frac{7}{20} \cdot 2\frac{4}{5}$

12.  $\frac{1}{10} \cdot 10$

13.  $\frac{1}{6} \div \frac{1}{2}$

14.  $\frac{7}{8} \div \frac{7}{8}$

15.  $\frac{9}{10} \div \frac{3}{5}$

16.  $\frac{3}{4} \div \frac{5}{8}$

17.  $18 \div \frac{2}{3}$

18.  $7\frac{1}{2} \div 2\frac{1}{10}$

19.  $6\frac{3}{7} \div 3$

20.  $1\frac{3}{25} \div \frac{1}{5}$

21. **AREA** Find the area of a rectangular court that is  $21\frac{3}{5}$  meters long and  $13\frac{3}{4}$  meters wide.

22. **CARPENTRY** How many  $1\frac{1}{4}$ -foot pieces can you cut from a piece of wood that is 20 feet long?

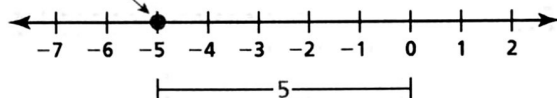
# Operations with Integers

## Adding and Subtracting Integers

The **absolute value** of an integer is the distance between the number and 0 on a number line. The absolute value of a number  $x$  is written as  $|x|$ .

**Example 1** Find the absolute value of  $-5$ .

Graph  $-5$  on a number line.



The distance between  $-5$  and 0 is 5.

► So,  $|-5| = 5$ .

### Rules for Adding and Subtracting Integers

<b>Adding:</b>	To add integers with the <i>same</i> sign, add the absolute values of the integers. Then use the common sign.  To add integers with <i>different</i> signs, subtract the lesser absolute value from the greater absolute value. Then use the sign of the integer with the greater absolute value.
<b>Subtracting:</b>	To subtract an integer, add its opposite.

**Example 2** Find (a)  $-3 + (-8)$  and (b)  $-9 + 6$ .

a.  $-3 + (-8) = -11$  Add  $|-3|$  and  $|-8|$ .  
Use the common sign.

► The sum is  $-11$ .

b.  $-9 + 6 = -3$   $|-9| > |6|$ . So, subtract  $|6|$  from  $|-9|$ .  
Use the sign of  $-9$ .

► The sum is  $-3$ .

**Example 3** Find (a)  $5 - (-12)$  and (b)  $1 - 7$ .

a.  $5 - (-12) = 5 + 12$  Add the opposite of  $-12$ .  
 $= 17$  Add.

► The difference is 17.

b.  $1 - 7 = 1 + (-7)$  Add the opposite of 7.  
 $= -6$  Add.

► The difference is  $-6$ .

**Example 4** Simplify  $|-14 - (-10)|$ .

$-14 - (-10) = |-14 + 10|$  Add the opposite of  $-10$ .  
 $= |-4|$  Add.  
 $= 4$  Find the absolute value.

► So,  $|-14 - (-10)| = 4$ .

# Operations with Integers

## Multiplying and Dividing Integers

### Rules for Multiplying and Dividing Integers

**Multiplying and Dividing:** The product or quotient of two integers with the *same* sign is *positive*.  
The product or quotient of two integers with *different* signs is *negative*.

**Example 5** Find (a)  $-7 \cdot (-1)$  and (b)  $-9 \cdot 4$ .

a.  $-7 \cdot (-1) = 7$  The integers have the same sign,  
so the product is positive.

► The product is 7.

b.  $-9 \cdot 4 = -36$  The integers have different signs,  
so the product is negative.

► The product is  $-36$ .

**Example 6** Find (a)  $18 \div (-2)$  and (b)  $-25 \div (-5)$ .

a.  $18 \div (-2) = -9$  The integers have different signs,  
so the quotient is negative.

► The quotient is  $-9$ .

b.  $-25 \div (-5) = 5$  The integers have the same sign,  
so the quotient is positive.

► The quotient is 5.

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Find the absolute value.

1.  $|13|$

2.  $|-8|$

3.  $|0|$

4.  $|-297|$

Evaluate.

5.  $5 + (-11)$

6.  $4 - 9$

7.  $-15 + (-10)$

8.  $9 + (-6)$

9.  $0 - (-50)$

10.  $-8 + 20$

11.  $-11 - 11$

12.  $-14 + 0$

13.  $20 - (-21)$

14.  $-34 - (-25)$

15.  $-8 + (-3) + 6$

16.  $1 + 7 - 9$

Simplify the expression.

17.  $|-15 - 9|$

18.  $|18 - (-11)|$

19.  $|-14 + 17|$

20.  $|-24 - (-19)|$

Evaluate.

21.  $-8 \cdot 25$

22.  $-33 \div (-3)$

23.  $-13(-1)$

24.  $-24 \div 4$

25.  $0(-4)$

26.  $-15(8)$

27.  $\frac{0}{-12}$

28.  $-1(-1)$

29.  $\frac{-16}{-1}$

30.  $240 \div (-8)$

31.  $5 \cdot (-7) \cdot (-4)$

32.  $12 \div (-3) \cdot 2$

33. **ELEVATION** The highest elevation in California is 14,494 feet, on Mount Whitney. The lowest elevation in California is  $-282$  feet in Death Valley. Find the range of elevations in California.

34. **GOLF** The table shows a golfer's score for each round of a tournament. Find the golfer's total score and the golfer's mean score per round.

	Round 1	Round 2	Round 3
Score	$-3$	$-4$	$+1$

# Operations with Rational Numbers

To add, subtract, multiply, or divide rational numbers, use the same rules for signs as you used for integers.

**Example 1** Find (a)  $-\frac{5}{6} + \frac{2}{3}$  and (b)  $7.3 - (-4.8)$ .

a. Write the fractions with the same denominator, then add.

$$-\frac{5}{6} + \frac{2}{3} = -\frac{5}{6} + \frac{4}{6} = \frac{-5 + 4}{6} = \frac{-1}{6} = -\frac{1}{6}$$

b. To subtract a rational number, add its opposite.

$$7.3 - (-4.8) = 7.3 + 4.8 = 12.1 \quad \text{The opposite of } -4.8 \text{ is } 4.8.$$

**Example 2** Find (a)  $2.25 \cdot 8$ , (b)  $-2.25 \cdot (-8)$ , and (c)  $-2.25 \cdot 8$ .

a.  $2.25 \cdot 8 = 18$

b.  $-2.25 \cdot (-8) = 18$

c.  $-2.25 \cdot 8 = -18$

**Example 3** Find  $-\frac{4}{9} \div \frac{3}{4}$ .

To divide by a fraction, multiply by its reciprocal.

$$-\frac{4}{9} \div \frac{3}{4} = -\frac{4}{9} \cdot \frac{4}{3} = -\frac{4 \cdot 4}{9 \cdot 3} = -\frac{16}{27} \quad \text{The reciprocal of } \frac{3}{4} \text{ is } \frac{4}{3}.$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Add, subtract, multiply, or divide.

1.  $-7.5 + 3.8$

2.  $-18.3 + (-6.7)$

3.  $0.6 - 0.85$

4.  $6.13 - (-2.82)$

5.  $-6 \cdot 4.75$

6.  $-3.2 \cdot (-4.8)$

7.  $-1.8 \div (-9)$

8.  $3.6 \div (-1.5)$

9.  $-\frac{1}{6} + \frac{5}{6}$

10.  $-\frac{7}{10} + \left(-\frac{3}{5}\right)$

11.  $\frac{4}{9} - \frac{2}{3}$

12.  $-\frac{5}{6} - \frac{1}{4}$

13.  $-\frac{3}{2} \cdot \left(-\frac{1}{8}\right)$

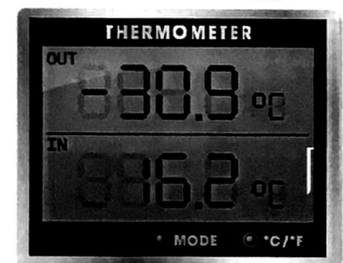
14.  $-\frac{3}{4} \cdot \frac{7}{12}$

15.  $\frac{5}{8} \div \left(-\frac{1}{4}\right)$

16.  $-\frac{4}{7} \div \frac{2}{5}$

**17. TEMPERATURE** The temperature at midnight is shown. The outside temperature decreases  $2.3^{\circ}\text{C}$  over the next two hours. What is the outside temperature at 2 A.M.?

**18. SNOWFALL** In January, a city's snowfall was  $\frac{5}{8}$  foot below the historical average. In February, the snowfall was  $\frac{3}{4}$  foot above the historical average. Was the city's snowfall in the two-month period above or below the historical average? By how much?



# Evaluating Algebraic Expressions

An **algebraic expression** is an expression that may contain numbers, operations, and one or more symbols. A symbol that represents one or more numbers is called a **variable**. To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

**Example 1** Evaluate each expression when  $x = 3$ .

a.  $5x + 7$

$$\begin{aligned} 5x + 7 &= 5(3) + 7 && \text{Substitute 3 for } x. \\ &= 15 + 7 && \text{Multiply.} \\ &= 22 && \text{Add.} \end{aligned}$$

b.  $14 - x^2$

$$\begin{aligned} 14 - x^2 &= 14 - 3^2 && \text{Substitute 3 for } x. \\ &= 14 - 9 && \text{Evaluate power.} \\ &= 5 && \text{Subtract.} \end{aligned}$$

c.  $2x^2 - 8x + 4$

$$\begin{aligned} 2x^2 - 8x + 4 &= 2(3)^2 - 8(3) + 4 && \text{Substitute 3 for } x. \\ &= 2(9) - 8(3) + 4 && \text{Evaluate power.} \\ &= 18 - 24 + 4 && \text{Multiply.} \\ &= -2 && \text{Simplify.} \end{aligned}$$

**Example 2** Evaluate each expression when  $x = -2$  and  $y = 6$ .

a.  $7x - 5y$

$$\begin{aligned} 7x - 5y &= 7(-2) - 5(6) \\ &= -14 - 30 \\ &= -44 \end{aligned}$$

b.  $x^2 - 2xy + y^2$

$$\begin{aligned} x^2 - 2xy + y^2 &= (-2)^2 - 2(-2)(6) + 6^2 \\ &= 4 - 2(-2)(6) + 36 \\ &= 4 - (-24) + 36 \\ &= 64 \end{aligned}$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Evaluate the expression when  $x = 2$  and  $y = -3$ .

1.  $3x + 10$

2.  $14 - 2y$

3.  $5 - y^2$

4.  $4x^2 + 9$

5.  $y^2 + 8y - 4$

6.  $-3x^2 - x + 7$

7.  $0.75x - 4x - 1.5$

8.  $3(y + 8 - 4y)$

9.  $2x + 3y$

10.  $6y - 5x$

11.  $4x^2 + 3y$

12.  $x^2 - y^2$

13.  $y - x + y^2$

14.  $x^2y^2 + xy$

15.  $\frac{x+y}{y-x}$

16.  $\frac{2x+y}{xy}$

Copy and complete the table.

17.

$x$	0	1	2	3	4
$3x - 2$					

18.

$x$	-2	-1	0	1	2
$-4x + 1$					

**19. MONEY** You earn  $8x + 7y$  dollars for working  $x$  hours at a restaurant and  $y$  hours at a bus station. How much do you earn for working 12 hours at the restaurant and 16 hours at the bus station?



# Properties of Equality

## Addition Property of Equality

**Words** When you add the same number to each side of an equation, the two sides remain equal.

**Numbers**  $6 + 4 = 6 + 4$   
 $10 = 10$

**Algebra**  $x - 5 + 5 = 3 + 5$   
 $x = 8$

## Subtraction Property of Equality

**Words** When you subtract the same number from each side of an equation, the two sides remain equal.

**Numbers**  $7 - 2 = 7 - 2$   
 $5 = 5$

**Algebra**  $y + 3 - 3 = 1 - 3$   
 $y = -2$

## Multiplication Property of Equality

**Words** When you multiply each side of an equation by the same nonzero number, the two sides remain equal.

**Numbers**  $\frac{6}{3} \cdot 3 = 2 \cdot 3$   
 $6 = 6$

**Algebra**  $\frac{z}{3} \cdot 3 = 2 \cdot 3$   
 $z = 6$

## Division Property of Equality

**Words** When you divide each side of an equation by the same nonzero number, the two sides remain equal.

**Numbers**  $6 \cdot 2 \div 2 = 12 \div 2$   
 $6 = 6$

**Algebra**  $\frac{2w}{2} = \frac{12}{2}$   
 $w = 6$

**Example 1** Solve each equation. Tell which algebraic property of equality you used.

a.  $c - 3 = -2$

$c - 3 + 3 = -2 + 3$  Addition Property of Equality  
 $c = 1$  Simplify.

► The solution is  $c = 1$ . The property is the Addition Property of Equality.

b.  $\frac{d}{5} = 7$

$\frac{d}{5} \cdot 5 = 7 \cdot 5$  Multiplication Property of Equality  
 $d = 35$  Simplify.

► The solution is  $d = 35$ . The property is the Multiplication Property of Equality.

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Solve the equation. Tell which algebraic property of equality you used.

1.  $h - 6 = 2$

2.  $\frac{j}{3} = 9$

3.  $k + 8 = -9$

4.  $4m = 12$

5.  $n + 2 = 6$

6.  $\frac{p}{6} = -2$

7.  $q - 3 = -8$

8.  $8r = 48$

9.  $s + 9 = 5$

10.  $6t = 48$

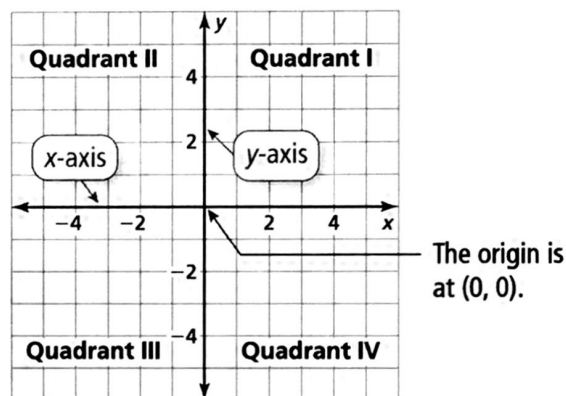
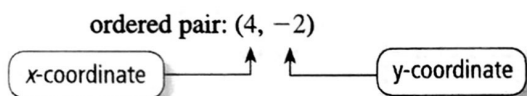
11.  $w + 3 = 29$

12.  $\frac{z}{7} = 7$

# The Coordinate Plane

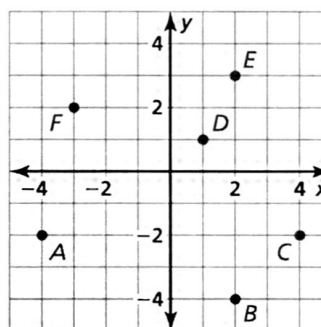
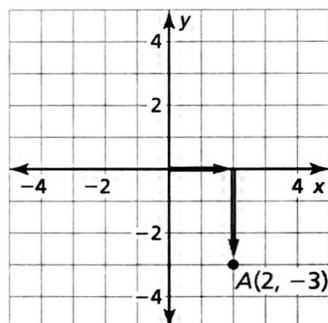
A **coordinate plane** is formed by the intersection of a horizontal number line and a vertical number line. The number lines intersect at the **origin** and separate the coordinate plane into four regions called **quadrants**.

An **ordered pair** is used to locate a point in a coordinate plane.



**Example 1** Plot the point  $A(2, -3)$  in a coordinate plane. Describe the location of the point. **Example 2** What ordered pair corresponds to point A?

Start at the origin. Move 2 units right and 3 units down. Then plot the point. The point is in Quadrant IV.



Point A is 4 units to the left of the origin and 2 units down. So, the x-coordinate is  $-4$  and the y-coordinate is  $-2$ .

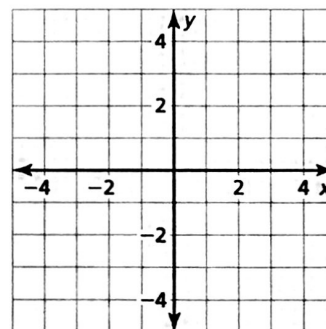
► The ordered pair  $(-4, -2)$  corresponds to point A.

## Practice

Plot the ordered pair in a coordinate plane. Describe the location of the point.

1.  $A(1, 3)$
2.  $B(-2, 2)$
3.  $C(2, -4)$
4.  $D(1, -1)$
5.  $E(-4, -2.5)$
6.  $F(-3, 0)$
7.  $G(0, 1)$
8.  $H(4, \frac{1}{2})$

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).



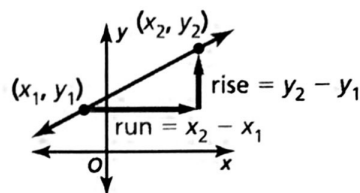
Use the graph in Example 2 to answer the questions.

9. What ordered pair corresponds to point C?
10. What ordered pair corresponds to point F?

# Slope of a Line

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line. If a line in the coordinate plane passes through points  $(x_1, y_1)$  and  $(x_2, y_2)$ , then the slope  $m$  is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



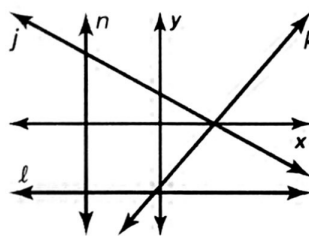
## Slopes of Lines in the Coordinate Plane

**Negative slope:** falls from left to right, as in line  $j$

**Positive slope:** rises from left to right, as in line  $k$

**Zero slope (slope of 0):** horizontal, as in line  $\ell$

**Undefined slope:** vertical, as in line  $n$



**Example 1** Find the slope of the line shown.

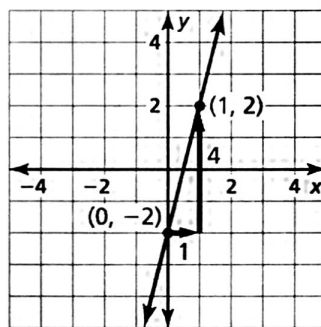
Let  $(x_1, y_1) = (0, -2)$  and  $(x_2, y_2) = (1, 2)$ .

$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-2)}{1 - 0} \\ &= 4 \end{aligned}$$

Write formula for slope.

Substitute.

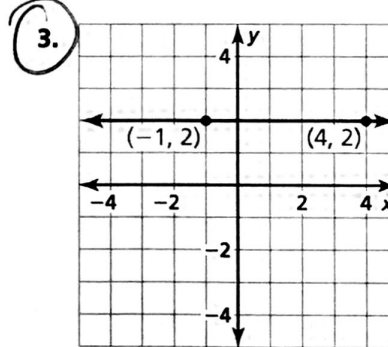
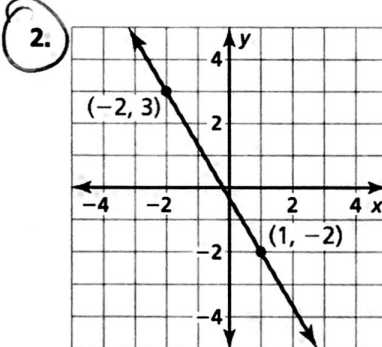
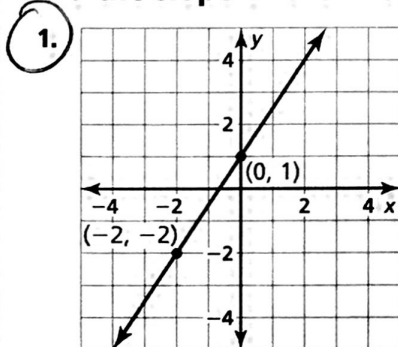
Simplify.



## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Find the slope of the line.



# Adding and Subtracting Polynomials

To add polynomials, add like terms. You can use a vertical or a horizontal format.

## Example 1 Find each sum.

a.  $(5x^2 + 3x - 7) + (x^2 + 2)$

Use a vertical format. Align like terms vertically and add.

$$\begin{array}{r} 5x^2 + 3x - 7 \\ + \quad x^2 \quad + 2 \\ \hline 6x^2 + 3x - 5 \end{array}$$

b.  $(-4x^3 - x + 1) + (2x^2 + 8x - 9)$

Use a horizontal format. Group like terms and simplify.

$$\begin{aligned} (-4x^3 - x + 1) + (2x^2 + 8x - 9) &= (-4x^3 + 2x^2) + (-x + 8x) + (1 - 9) \\ &= -2x^3 + 7x - 8 \end{aligned}$$

To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by  $-1$ .

## Example 2 Find each difference.

a.  $(6x^3 - 2x - 5) - (-x^3 + 3x^2 + 4)$

Use a vertical format. Align like terms vertically and subtract.

$$\begin{array}{r} 6x^3 \quad \quad - 2x - 5 \\ - (-x^3 + 3x^2 \quad + 4) \\ \hline 6x^3 \quad \quad - 2x - 5 \\ + \quad x^3 - 3x^2 \quad - 4 \\ \hline 7x^3 - 3x^2 - 2x - 9 \end{array}$$

b.  $(5x^2 + 7x - 3) - (4x^2 - \quad + 2x - 1)$

Use a horizontal format. Group like terms and simplify.

$$\begin{aligned} (5x^2 + 7x - 3) - (4x^2 - \quad + 2x - 1) &= 5x^2 + 7x - 3 - 4x^2 + 2x + 1 \\ &= (5x^2 - 4x^2) + (7x + 2x) + (-3 + 1) \\ &= x^2 + 9x - 2 \end{aligned}$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Find the sum or difference.

1.  $(-8x + 2) + (-10x - 7)$

2.  $(x^3 + 9x) - (4x^3 - x)$

3.  $(x^2 - 2x - 6) + (6x^2 + x + 8)$

4.  $(2x^3 + 5x^2 - x) + (x^3 - 10x^2 + 5x)$

5.  $(-7x^3 - x^2 + 10) - (3x^2 + 2x - 2)$

6.  $(x^3 + 8x + 3) - (-x^3 + 2x^2 - 5)$

7.  $(x^2 + 4x + 1) + (-x^2 - 1)$

8.  $(3x^3 - 2x^2) - (5x^3 - x^2 - x)$

9.  $(2x^3 - 5) - (-8x^2 - 5x)$

10.  $(-x^2 - 4) + (x^3 - 4x^2)$

# The Distributive Property

To multiply a sum or difference by a number, multiply each number in the sum or difference by the number outside the parentheses, then evaluate.

Distributive Property	
With addition: $5(7 + 3) = 5(7) + 5(3)$	$a(b + c) = a(b) + a(c)$
With subtraction: $5(7 - 3) = 5(7) - 5(3)$	$a(b - c) = a(b) - a(c)$

**Example 2** Simplify each expression.

a.  $6(x + 9)$

$$\begin{aligned} 6(x + 9) &= 6(x) + 6(9) \\ &= 6x + 54 \end{aligned}$$

b.  $10(12 + z + 7)$

$$\begin{aligned} 10(12 + z + 7) &= 10(12) + 10(z) + 10(7) \\ &= 120 + 10z + 70 \\ &= 10z + 190 \end{aligned}$$

c.  $16(8w - 3)$

$$\begin{aligned} 16(8w - 3) &= 16(8w) - 16(3) \\ &= 128w - 48 \end{aligned}$$

d.  $5(4m - 3n - 1)$

$$\begin{aligned} 5(4m - 3n - 1) &= 5(4m) - 5(3n) - 5(1) \\ &= 20m - 15n - 5 \end{aligned}$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

**Evaluate.**

1.  $25(7 + 11)$

2.  $4(13 - 5)$

3.  $9(16 + 7 - 8)$

4.  $-4(10 - 9 - 6)$

**Simplify the expression.**

5.  $4(y + 7)$

6.  $-2(z + 5)$

7.  $5(b - 11)$

8.  $-8(d - 1)$

9.  $12(4a + 13)$

10.  $9(20 + 17m)$

11.  $11(2k - 11)$

12.  $-7(-2n - 9)$

13.  $3(x + 4 + 9)$

14.  $6(25 + 6z + 10)$

15.  $8(p - 6 - 5)$

16.  $-10(4 + v - 1)$

17.  $7(2x + 7 + 9y)$

18.  $-4(4r - s + 17)$

19.  $-3(-12 - 3d - 8)$

20.  $2 - 6(2n - 9)$

21.  $1.5(6c + 10d + 3)$

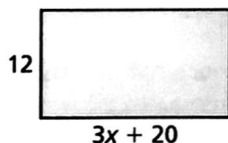
22.  $\frac{3}{4}\left(q + \frac{1}{6} + \frac{7}{8}\right)$

23.  $-2.4(5h - 10 + 4)$

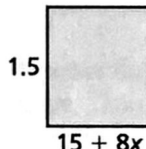
24.  $0.5(2.6x + 5.8)$

**Write and simplify an expression for the area of the rectangle.**

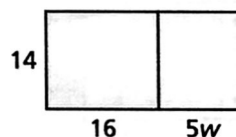
7.



8.



9.



# Powers and Exponents

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.

$$\begin{array}{ccc} \text{base} & & \text{exponent} \\ \downarrow & & \downarrow \\ \left(\frac{2}{5}\right)^3 = \underbrace{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}}_{\text{power}} & & \underbrace{\frac{2}{5} \text{ is used as factor 3 times}}_{\text{exponent}} \end{array}$$

**Example 1 Write each product using exponents.**

a.  $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

Because  $-9$  is used as a factor 5 times, its exponent is 5.

► So,  $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^5$ .

b.  $\pi \cdot \pi \cdot h \cdot h \cdot h$

Because  $\pi$  is used as a factor 2 times, its exponent is 2. Because  $h$  is used as a factor 3 times, its exponent is 3.

► So,  $\pi \cdot \pi \cdot h \cdot h \cdot h = \pi^2 h^3$ .

**Example 2 Evaluate each expression.**

a.  $(-5)^4$

$$\begin{aligned} (-5)^4 &= (-5) \cdot (-5) \cdot (-5) \cdot (-5) \\ &= 625 \end{aligned}$$

Write as repeated multiplication.

Simplify.

b.  $-5^4$

$$\begin{aligned} -5^4 &= -(5 \cdot 5 \cdot 5 \cdot 5) \\ &= -625 \end{aligned}$$

Write as repeated multiplication.

Simplify.

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

**Write the product using exponents.**

1.  $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

2.  $\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right)$

3.  $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$

4.  $2.5 \cdot 2.5 \cdot b \cdot b \cdot b \cdot b$

5.  $(-n) \cdot (-n) \cdot (-n) \cdot (-n)$

6.  $(-12) \cdot (-12) \cdot v \cdot v \cdot v$

**Evaluate the expression.**

7.  $10^4$

8.  $-15^2$

9.  $\left(\frac{3}{4}\right)^3$

10.  $\left(-\frac{1}{2}\right)^5$

11. **VOLUME** Write an expression involving a power that represents the volume (in cubic centimeters) of the die shown. Then find the volume.

